

Eton College King's Scholarship Examination 2014

(One and a half hours)

MATHEMATICS A

Answer Question 1 and as many of the other five questions as you can.

Question 1 is worth 50 marks. All other questions are worth 10 marks each.

*Show all of your working. The use of calculators is **not** permitted.*

Do not turn over until told to do so.

1. This question is compulsory.

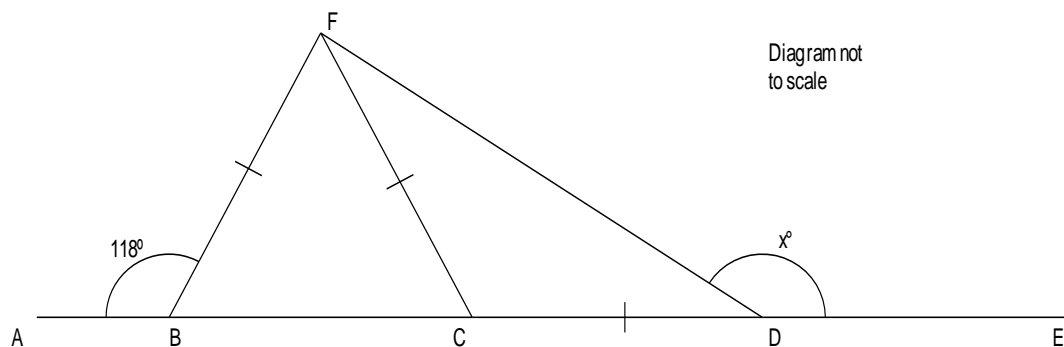
(a) Calculate the following, leaving your answers as simplified fractions:

(i) $4\frac{1}{12} + 5\frac{5}{48}$ [3]

(ii) $\frac{34}{35} \times \frac{42}{51}$ [3]

(b) Calculate 115% of 80% of £15. [3]

(c) In the diagram below, ABCDE is a straight line and all three lengths BF, CF and CD are equal. Given that angle ABF = 118° , calculate the angle EDF (marked as x° in the diagram). [2]



(d) What do I subtract from $4 - 5x$ to get $-4x + 5$? [3]

(e) Solve the following equations, giving your answers as simplified fractions:

(i) $\frac{3}{28}(a+5) = \frac{5}{8}$ [3]

(ii) $1 - \frac{1}{3}b = b - 1$ [3]

(f) Solve the simultaneous equations:

$$11x - 8y = 18$$

$$7x + 4y = 16$$

[4]

(g) Solve the following inequalities:

(i) $\frac{2}{3}x > 1 + \frac{1}{2}x$ [3]

(ii) $2x - 7 \geq 2 + 7x$ [3]

(h) Two circles have radii 0.4 cm and 1.2 cm. The area of the smaller circle is x and the total area of the two circles is y . What percentage of y is x ? [4]

- (i) The diagram shows a triangle ABC. The point P lies on the side BC and angle CPA is 90° . AB and AP are 4 cm and 2.4 cm respectively. PC is 1 cm.
- (i) Show that AC has length 2.6 cm. [3]
- (ii) Calculate the perimeter of the triangle ABC. [4]

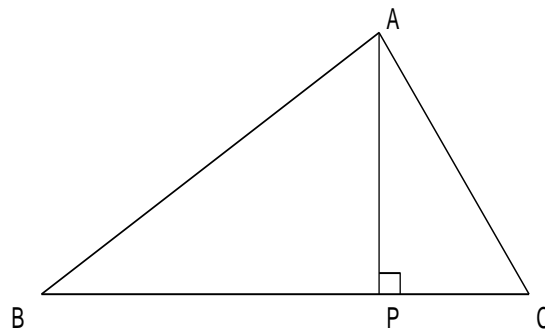


Diagram not to scale

- (j) Edward scores an average of 26 runs in his first three cricket matches. After his first five cricket matches, he scores an average of 27.8 runs. If he scored 9 more runs in his fifth match than he did in his fourth match, how many runs did he score in his fifth match? [4]

- (k) (i) By what do you multiply $5\frac{1}{4}$ to get 7? Give your answer as an exact fraction. [2]

- (ii) By what do you multiply $\frac{a}{2b}$ to get $\frac{ab}{4}$? [3]

2. (a) Show that $(a+2b)^2 = a^2 + 4ab + 4b^2$. [2]

(b) (i) Multiply out and simplify the expression $(a+2b)^2 + (2a-b)^2$. [3]

(ii) Hence calculate $1002^2 + 1999^2$. [2]

(c) Find two whole numbers n and m such that $n^2 + m^2 = 20000245$. [3]

3. (a) In the diagram below, the lines AB and ED are parallel.
Angles ABC, BCD and CDE are x° , y° and z° respectively.

Giving reasons, show that $x - y - z + 180 = 0$.

[4]

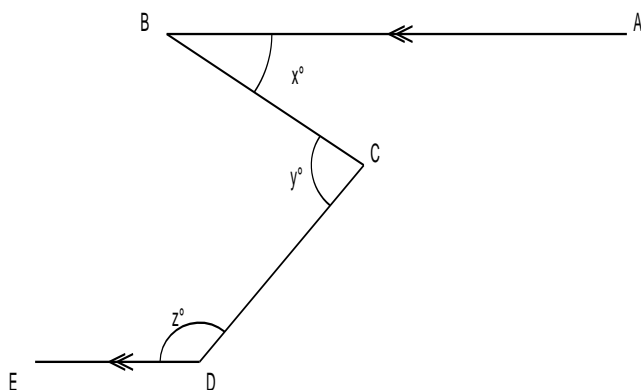


Diagram not to scale

- (b) The diagram below shows a parallelogram PQRS.
The two lengths PX and QX are equal.
The two angles SRX and QRX are equal.

(a) Giving reasons, show that triangle XQR is isosceles.

[2]

(b) Giving reasons, show that angle SXR is a right angle.

[4]

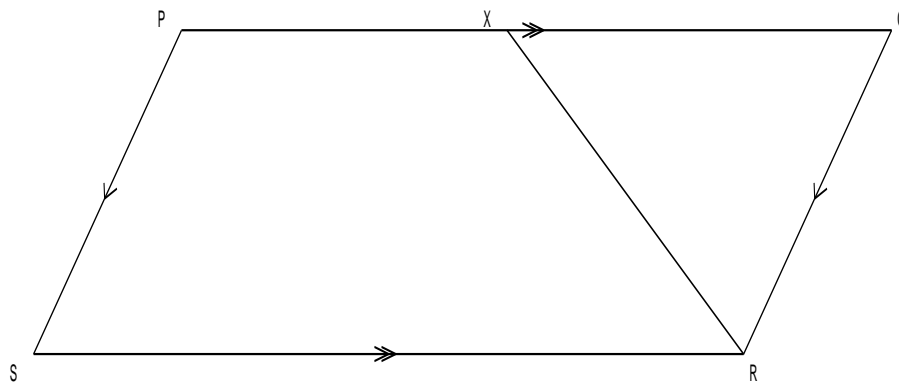


Diagram not to scale

4. You are given that $123456789 = 3803 \times 32463$.

(a) Write down the answers to the following calculations:

(i) 0.3803×32463000 [1]

(ii) $\frac{12345.6789}{380300}$ [2]

(iii) $\frac{12345.6789}{0.03803}$ [2]

(iv) $\frac{3803123456789}{3803}$ [2]

(b) Calculate the value of $\frac{38033803 \times 3246332463}{123456789}$. [3]

5. In this question, a word is defined to be a set of letters, each letter of which is either an L or an R, written down in order. A word is good if it does not contain 3 consecutive letters which are the same. Otherwise a word is bad. The length of a word is the number of letters in it.

For example, LRRL is a good word of length 4 and is a different good word to LRLR. LRRRL and LRLLLR are both bad words.

(a) Write down all the good words of length 4. [2]

(b) If I write down a good word which ends in LR, explain why I can make two different good words from it by adding a letter at the end. Would this be true if I started with a good word ending in LL? [2]

(c) Copy and complete the following table.

	Number of good words of length 4	Number of good words of length 5	Number of good words of length 6
Last two letters LL	2		
Last two letters RL	3		8
Last two letters RR	2		
Last two letters LR	3		

(e) How many good words are there of length 15? [2]

[4]

6. The number 12 has 6 factors (1, 2, 3, 4, 6 and 12).
- (a) Find the number of factors (you need not list them) for each of the following:
- (i) 75
 - (ii) 847 [2]
- (b) Explain clearly why $7^4 \times 11^3$ has exactly 20 factors. [2]
- (c) How many factors does $2^4 \times 3^2 \times 5 \times 7^3$ have? [2]
- (d) Explain why all square numbers have an odd number of factors. [2]
- (e) Find the smallest number with exactly 36 factors. [2]

[End of paper]